

component having $\lambda/\rho_{Al} = 7.4$, in proportions which increase with the voltage, at any rate, within a moderate range.

The extent of the work was not competent to throw doubt on the prevailing view that the bulk of the X-rays from a bulb worked under voltages such as obtain in medical practice consists of other than general heterogeneous radiation.

Summary.

The X-rays from a bulb excited by low voltages (10,000 to 50,000 volts) are rich in the characteristic radiation of the anticathode. In the case of iron, nickel and copper, the amount of K-radiation lies between 80 and 90 per cent. In the case of platinum the proportion of L-radiation is from 40 to 60 per cent. Evidence of characteristic radiations softer than the K and L-radiation has been obtained.

The writer has perforce had to content himself with little more than setting down observations. For the preparation of the diagrams he is greatly indebted to Mr. W. F. Higgins, B.Sc.

On Magnetic Inertia.

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In the course of some investigations with regard to the Earth's magnetism, it occurred to me that a magnetised body must possess magnetic inertia in virtue of its magnetisation, just as an electrified body possesses electric inertia. I am not aware that this has been considered before, nor have I seen any calculation of the amount to be expected.

The method of dimensions shows that the magnetic inertia of a body of magnetic moment m should be a numerical multiple of $m^2 a^{-3} C^{-2}$, where C is the velocity of radiation and a a linear dimension of the body (*e.g.*, the radius, in the case of a sphere).

The problem may be investigated and the numerical factor determined by the same general method used for electric inertia.* Consider a conducting sphere of radius a uniformly magnetised with magnetic moment m .

* 'Roy. Soc. Proc.,' A, vol. 77, p. 260 (1906), and 'Phil. Trans.,' A, vol. 210, p. 145 (1910).

(1) Let the sphere be magnetised parallel to the axis of x . When the sphere is at rest the magnetic forces are given by

$$\begin{aligned} \text{inside } (\alpha, \beta, \gamma) &= -ma^{-3}(1, 0, 0), \\ \text{outside } (\alpha, \beta, \gamma) &= -mr^{-3}(1, 0, 0) + 3m\alpha r^{-5}(x, y, z). \end{aligned}$$

Thus the tangential component of (α, β, γ) is continuous at $r = a$, and the apparent surface density is $\sigma = 3m\alpha a^{-4}/4\pi$.

Now, let the sphere move under the action of a mechanical force so that the displacement along the axis of x at any time is ξ . The appropriate forms to be assumed for the field outside are (*cf. loc. cit.*):

$$\begin{aligned} (\alpha, \beta, \gamma) &= -mr^{-3}(1, 0, 0) + 3m\alpha r^{-5}(x, y, z) \\ &\quad + C\alpha r^{-5}(1, 0, 0)(r^3\chi''' + 3r^2\chi'' + 6r\chi' + 6\chi + 6\xi m/C) \\ &\quad + Cr^{-5}(x, y, z)(r^2\chi'' + 3r\chi' + 3\chi + 3\xi m/C) \\ &\quad - C\alpha^2 r^{-7}(x, y, z)(r^3\chi''' + 6r^2\chi'' + 15r\chi' + 15\chi + 15\xi m/C) \\ (X, Y, Z) &= C\alpha r^{-5}(0, -z, y)(r^2\chi''' + 3r\chi'' + 3\chi'), \end{aligned}$$

where χ stands for $\chi(Ct - r)$.

Hence the electrodynamic force is

$$(X', Y', Z') = C\alpha r^{-5}(0, -z, y)(r^2\chi''' + 3r\chi'' + 3\chi' + 3\xi m/C^2).$$

We are at liberty to assume that inside

$$(\alpha, \beta, \gamma) = -ma^{-3}(1, 0, 0),$$

but we must suppose that the surface $r = a$ is the seat of electric currents given by

$$(u, v, w) = C\alpha a^{-6}(0, -z, y)P.$$

Now this current is determined by the discontinuity of the tangential component of (α, β, γ) at $r = a$.

$$\text{Hence } 4\pi P = -(a^3\chi''' + 3a^2\chi'' + 6a\chi' + 6\chi + 6\xi m/C). \quad (1)$$

The tangential component of X', Y', Z' vanishes at $r = a$ if

$$3\xi m + C^2(a^3\chi''' + 3a\chi'' + 3\chi') = 0,$$

which is satisfied by

$$3\xi m + C(a^3\chi'' + 3a\chi' + 3\chi) = 0. \quad (2)$$

In virtue of (2) the normal magnetic force is found to be, as before,

$$\begin{aligned} \text{outside } N_o &= 2m\alpha a^{-4} \\ \text{inside } N_i &= -m\alpha a^{-4}, \end{aligned}$$

so that the apparent surface density $\sigma = 3m\alpha a^{-4}/4\pi$ is unaltered by the motion.

The magnetic force in the direction of x at $r = a$ is

$$\text{outside } \alpha_o = -ma^{-3} + 3mx^2a^{-5} + Cxa^{-5}(a^3\chi''' + a^2\chi'') - Cx^3a^{-7}(a^3\chi''' + a^2\chi'')$$

and inside

$$\alpha_i = -ma^{-3}.$$

Thus

$$\frac{1}{2}(\alpha_o + \alpha_i) = -ma^{-3} + \frac{3}{2}mx^2a^{-5} + \frac{1}{2}Cxa^{-5}(1 - x^2/a^2)(a^3\chi''' + a^2\chi'').$$

Hence the force in the direction of x due to the magnetic force acting on the apparent surface density is

$$= \frac{1}{2} \int (\alpha_o + \alpha_i) \sigma 2\pi a^2 \sin \theta d\theta$$

and this reduces to

$$\frac{1}{5}mCa^{-3}(a\chi''' + \chi'').$$

The force in the direction of x due to the magnetic force on the surface current is

$$\frac{1}{2} \int (v\gamma_o - w\beta_o) 2\pi a^2 \sin \theta d\theta$$

and this reduces to

$$\frac{1}{5}mCa^{-3}(a\chi''' + \chi'').$$

Thus the total force in the direction of x due to radiation

$$= \frac{2}{5}mCa^{-3}(a\chi''' + \chi'').$$

But in a uniformly accelerated motion $\chi''' = 0$, and from (2)

$$C\chi'' = -m\ddot{\xi}/C^2$$

so that the force is

$$-\frac{2}{5}m^2a^{-3}C^2\ddot{\xi}.$$

Hence the magnetic inertia of the sphere for uniform acceleration in the direction of its magnetic axis is

$$\frac{2}{5}m^2a^{-3}C^{-2}.$$

(2) Let the sphere be magnetised parallel to the axis of z . When the sphere is at rest the magnetic forces are given by

$$\text{inside } (\alpha, \beta, \gamma) = -ma^{-3}(0, 0, 1)$$

$$\text{outside } (\alpha, \beta, \gamma) = -mr^{-3}(0, 0, 1) + 3mzr^{-5}(x, y, z).$$

Thus the tangential component of (α, β, γ) is continuous at $r = a$, and the apparent surface density is $\sigma = 3mza^{-4}/4\pi$.

Now, let the sphere move under the action of a mechanical force so that the displacement along the axis of x at any time is ξ . The appropriate forms to be assumed for the forces outside are

$$\begin{aligned} (\alpha, \beta, \gamma) = & -mr^{-3}(0, 0, 1) + 3mzr^{-5}(x, y, z) \\ & + Czr^{-5}(1, 0, 0)(r^3\chi''' + 2r^2\chi'' + 3r\chi' + 3\chi + 3\xi m/C) \\ & + Cxr^{-5}(0, 0, 1)(r^2\chi'' + 3r\chi' + 3\chi + 3\xi m/C) \\ & - Cxzr^{-7}(x, y, z)(r^3\chi''' + 6r^2\chi'' + 15r\chi' + 15\chi + 15\xi m/C), \end{aligned}$$

$$\begin{aligned} (X, Y, Z) = & Cr^{-3}(0, 1, 0)(r\chi'' + \chi') \\ & - Czr^{-5}(0, z, -y)(r^2\chi''' + 3r\chi'' + 3\chi'). \end{aligned}$$

Hence

$$(X', Y', Z') = Cr^{-3}(0, 1, 0)(r\chi'' + \chi' + \xi m/C^2) \\ - Czr^{-5}(0, z, -y)(r^2\chi''' + 3r\chi'' + 3\chi' + 3\xi m/C^2).$$

Inside we may take

$$(\alpha, \beta, \gamma) = -ma^{-3}(0, 0, 1)$$

and assume currents at the surface $r = a$, viz.,

$$(u, v, w) = Cza^{-6}(0, -z, y)P.$$

Thus the tangential component of X', Y', Z' will vanish at $r = a$ if

$$a\chi'' + \chi' + \xi m/C^2 = 0$$

and

$$a^2\chi''' + 3a\chi'' + 3\chi' + 3\xi m/C^2 = 0.$$

Again the tangential discontinuity of α, β, γ will measure the surface current if

$$4\pi P = -(a^3\chi''' + 2a^2\chi'' + 3a\chi' + 3\chi + 3\xi m/C)$$

and

$$3\xi m + C(a^2\chi'' + 3a\chi' + 3\chi) = 0.$$

These conditions reduce to two, viz.,

$$4\pi P = -a^2\chi'', \quad (3)$$

$$3\xi m + C(a^2\chi'' + 3a\chi' + 3\chi) = 0, \quad (4)$$

provided $\chi''' = 0$.

Under these conditions the normal component of magnetic force outside reduces to

$$N_o = 2mza^{-4}$$

and inside

$$N_i = -mza^{-4},$$

so that the apparent surface density remains $\sigma = 3za^{-4}/4\pi$.

The magnetic force in the direction of x at $r = a$ is

$$\text{outside } \alpha_o = 3mzxa^{-5} + Cza^{-5}(1 - x^2/a^2)a^2\chi'',$$

$$\text{inside } \alpha_i = 0,$$

$$\text{therefore, } \frac{1}{2}(\alpha_o + \alpha_i) = \frac{3}{2}mzxa^{-5} + \frac{1}{2}Cza^{-5}(1 - x^2/a^2)a^2\chi''.$$

Thus the force in the direction x due to the magnetic force acting on the apparent surface density is

$$\frac{1}{2} \int \sigma (\alpha_o + \alpha_i) a^2 \sin \theta d\theta d\phi,$$

which reduces to

$$\frac{2}{5}mCa^{-3}\chi''.$$

The force in the direction x due to the magnetic force acting on the surface current is

$$= \frac{1}{2} \int \{v(\gamma_o + \gamma_i) - w\beta_o\} a^2 \sin \theta d\theta d\phi,$$

and this reduces to

$$\frac{2}{5}mCa^{-3}\chi''.$$

The total force in the direction of x due to radiation is thus

$$\frac{4}{5}mCa^{-3}\chi''.$$

Hence since $C\chi'' = -m\ddot{\xi}/C^2$ for a uniformly accelerated motion, the force is

$$-\frac{4}{5}m^2a^{-3}C^{-2}\ddot{\xi}.$$

Hence the magnetic inertia of the sphere for uniform acceleration at right angles to the axis of magnetisation is

$$\frac{4}{5}m^2a^{-3}C^{-2}.$$

We may now consider some applications of these results.

In the case of the Earth, the electric inertia, assuming a normal potential gradient at the surface of 1 volt per cm., comes to 2.1 gm. The magnetic moment of the Earth is, according to Gauss, 8.6×10^{25} C.G.S. magnetic units. Thus the magnetic inertia parallel to its magnetic axis works out to 1.3×10^4 gm. Thus the magnetic inertia is more important than the electric inertia, although it is still very small compared with the gravitational inertia, viz., 6×10^{27} gm.

The ratio of magnetic to gravitational inertia for the Earth is thus 2×10^{-24} .

Prof. Hale's measurements lead to the conclusion that the surface magnetic forces on the Sun are 100 times those at the Earth's surface, and as its density is but a quarter of that of the Earth, the corresponding ratio for the Sun would be about 8×10^{-20} .

The fact that the inertia along and perpendicular to the magnetic axis are different is of interest. For since the Earth's magnetic axis is inclined to the axis of rotation, and this again is inclined to the plane of the ecliptic, there will be a small diurnal variation of the effective inertia of the Earth, which may be expected to lead to small diurnal corrugations in the Earth's path. It seems, however, doubtful if this effect could be measured within any reasonable astronomical time.

The results are also of interest in the theory of electrons and magnetons.

The electronic charge is 1.7×10^{-20} (in electromagnetic units), while the magnetic moment of the magneton is 1.8×10^{-21} (in magnetic units). If we suppose that in each case the radius is 1.5×10^{-13} , we find that the electric inertia is 1.3×10^{-27} , and the magnetic inertia is 4×10^{-25} . Thus the magnetic inertia is more important than the electric inertia.

There are, however, difficulties of a serious nature in supposing that the radius of a magneton is as small as 1.5×10^{-13} , and in these 'Proceedings,' A, vol. 91, p. 156 (1915), I give reasons for thinking that the radius is more nearly 1.5×10^{-10} .

The electric inertia would then be 1.3×10^{-30} and the magnetic inertia 4×10^{-34} .

It may be inferred from the results for the inertia of a magnetic doublet that the like results hold for an electric doublet where m stands for the electric moment in electrostatic units.

An unpolarised body consisting of electric or magnetic doublets may be supposed to have the axes of the doublets orientated equally in all directions. On the average the inertia per doublet is $\frac{2}{3}m^2a^{-3}C^{-2}$, and thus there is a contribution to the inertia of such a body of amount $\frac{2}{3}Nm^2a^{-3}C^{-2}$ per unit volume, where N is the number of doublets per unit volume. We may form an estimate of the order of magnitude of this effect.

I have shown* that the dielectric constant of a gas is given by

$$K = 1 + 4\pi Nm^2M^{-1}\bar{v}^{-2},$$

where M is the mass of a particle and $\bar{v}$ the velocity of mean square. Clearly a similar form would hold for the magnetic permeability if the particles were magnetic doublets.

Boltzmann's value of $K-1$ for hydrogen is 26.5×10^{-5} , and this with the usual estimates gives the electric moment per molecule (m) about 3×10^{-19} and a about 6×10^{-10} . If these values are applicable to a solid for which N is about 10^{24} , the inertia per unit volume on account of the electric doublets would be of order 10^{-6} or 10^{-7} .

We have then an effect which may prove capable of measurement. It involves a new combination of the molecular quantities and is connected with the magnetic permeability or dielectric constant of the material.

Note.—Sir J. J. Thomson has pointed out to me that the magnetic inertia may be calculated in a simple way by the method which he used in his original calculation of electric inertia for a charged sphere.

I may also add that the rotation of a magnetised sphere may be examined by the method in the paper. I find that for rotation about the magnetic axis the moment of inertia is $\frac{2}{3}m^2a^{-1}C^{-2}$, and for rotation about an axis at right angles to the magnetic axis it is $\frac{4}{3}m^2a^{-1}C^{-2}$.

* 'Roy. Soc. Proc.,' A, vol. 69, p. 394.